Inductance Formula of a Square Spiral Inductor on Grounded Substrate by Duality and Synthetic Asymptote

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Abstract **This paper gives a simple CAD formula for inductance of square spiral inductor with grounded substrate, by duality and synthetic asymptote. The average error of the formula is less than 2%. The formula also gives good physical insights to the layout design of the inductor spiral.**

I. INTRODUCTION

Spiral inductors are often used in portable wireless communication equipment to satisfy the design requirements, such as, low cost, low supply voltage, low power dissipation, low noise, high frequency of operation, and low distortion. Many authors have treated the spiral inductors with different analyses. Most of these approaches were based on numerical techniques [1, 2], curve fitting, or empirical formulas.

In 1987, Chow and She [3] derived an inductance formula of spiral inductors in *free space* through the *duality* of $LC=\mu_0\varepsilon_0$. This early formula was inaccurate as it did not account for the gaps between the spirals. The reason was that the techniques for including the gap were not developed, and neither was an available software, accurate and fast enough for validation at that time.

This difficulty is now resolved with the novel analytical techniques of *synthetic asymptote* and the *analytical moment method*.

Synthetic asymptote has been used recently to generate formulas in microwave, a list available in [4]. Briefly, it is constructed from two known asymptotes at the two limits of a parameter, and adjusted for accuracy in between. The formulas obtained by synthetic asymptote technique are simple, accurate and therefore give good physical insights.

Analytical moment method is the moment matrix expressed in the variational form and then into an analytical formula. This formula is easily adjusted for plate perforations [4], including the gaps between spirals.

The formula of inductance of spiral inductor is derived in detail in this paper. The average error is less than 2%. The formulas of the stray capacitors of the spiral can be derived in a similar way.

II. CAD FORMULAS FOR INDUCTANCE OF SPIRAL INDUCTORS

Spiral inductors can be fabricated in many shapes, such as square, hexagonal, octagonal, and circular. In this initial paper, we only consider the square spiral inductor of Fig. 1. The inductor is completely specified by the number of turns N, the turn width W, the gap S between spirals, and any one of the following: d_{out}, the outermost dimension, d_{in}, the innermost dimension, or the *fill ratio* of the spiral inductor $\rho=(d_{out}-d_{in})/(d_{out}+d_{in})$. For a spiral inductor with grounded substrate, we need two more parameters: h, the substrate thickness, and ε_r, the dielectric constant.

In this paper, however, we consider only the formula of the inductance of the spiral and not the parasitic capacitances. Being of magnetic field, inductance is independent of the substrate dielectric. Therefore, the inductor may again be assumed in free space as in [3] but with the added spiral gaps and ground plane.

The derived inductance formula is verified against numerical results at low frequency [5]. That is, the frequency in the numerical computation will be low enough that the parasitic capacitance would not affect the reactance obtained and therefore the inductance value.

Fig. 1 shows the square spiral inductor with grounded substrate. The thickness of the inductor is considered zero, as the thickness normally has little effect on the inductance.

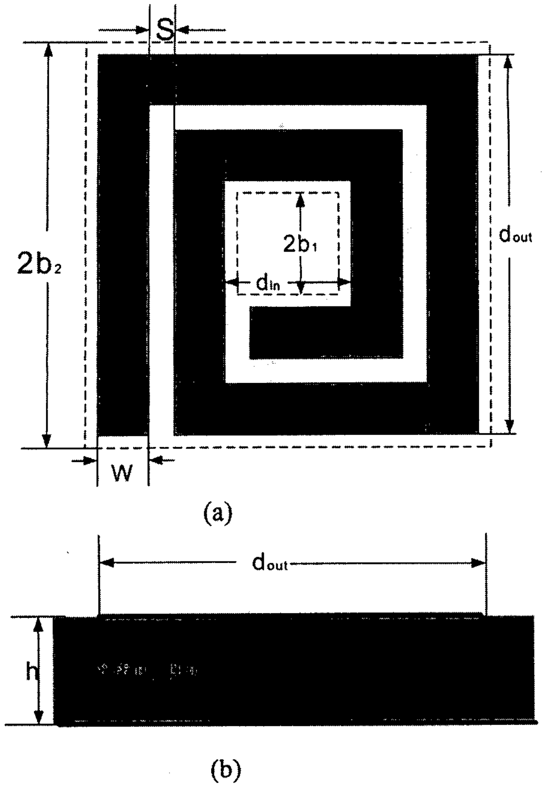

Fig. 1. The configuration of square spiral inductor: (a) top view; (b) side view.

A square spiral inductor may be divided into 4 trapezoids, as in Fig. 2a. Adjacent trapezoids have no mutual inductance as their currents are perpendicular. The two opposing trapezoids has opposite currents, therefore, they have mutual inductance.

A. The Far Asymptote of substrate thickness h

When the substrate is very thick, that is: $h \to \infty$, the *far* ground plane has little effect on the spiral inductor. This means that the spiral inductor is in a homogeneous free space.

Consider a solid trapezoidal quarter-plate in free space from Fig. 2a. Based on the "root of area" [4], the capacitance of this *solid* quarter-plate is:

$$C_{1/4solid} = c_{f1}\varepsilon_0\sqrt{8\pi A_{1/4}} \qquad (1)$$

where $A_{1/4} = b_2^2 - b_1^2$ with effective widths of the trapezoid $2b_2 = d_{out} + S$, $2b_1 = d_{in} + S$, and the

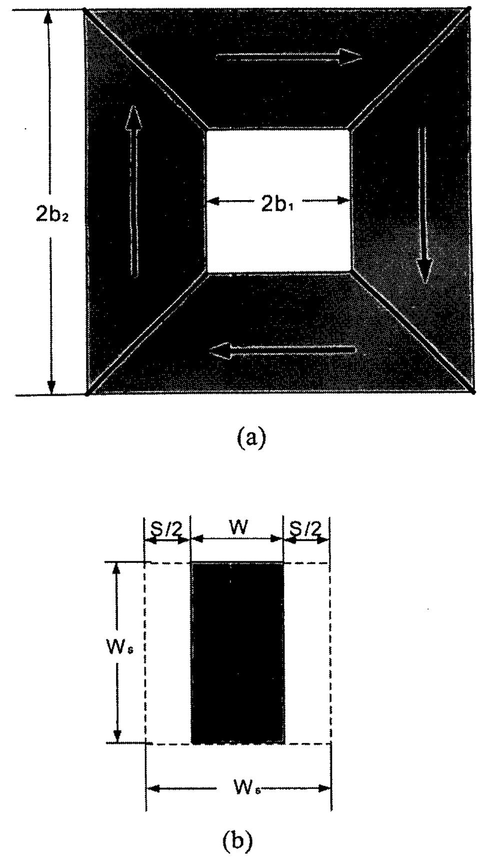

Fig. 2. The division of spiral inductor: (a) 4 trapezoids, (b) a square segment along the spiral.

shape factor c_{f1} of the trapezoidal plate is to be computed later in (4).

The area of each quarter of the spiral inductor is divided into $N \times M$ equal square segments of W_s^2. As shown in Fig. 2b, each segment has a conducting-area WW_s plus the surrounding gap-area. Here, N is the turn number of the spiral, and $M = (b_2 + b_1)/W_s$ with $W_s = W + S$, is the averaged sub-area number across a straight spiral arm.

A grid is formed with a small conducting sub-area deleted from each square segment on a solid-plate [4]. The shape of the sub-area itself has been shown to be unimportant. The empty gap-areas of Fig. 2b are also small conducting sub-areas deleted from a segment of the quarter-plate. Therefore we may say that the quarter-spiral of Fig. 1a is also a grid formed the solid quarter-plate in Fig. 2a.

The formula of the capacitance of a conducting grid has been derived from the analytical moment method (MoM) and the variational principle [4]. From this the capacitance of the quarter-spiral, *far* from the ground plane, can be written as:

$$C_{1/4far} = \frac{1}{\left[\frac{1}{c_{f1}\varepsilon_0\sqrt{8\pi A_{1/4}}} + \frac{\Delta p_{11} - \Delta p_{110}}{NM}\right] - \frac{1}{4\pi\varepsilon_0 r_0}} \tag{2}$$

where $r_0 = \frac{4(b_1^2 + b_1 b_2 + b_2^2)}{3(b_1 + b_2)}$ is the distance of the two centroids of the two opposing trapezoids. The terms in the bracket of the denominator are the self-potential of the quarter-spiral. The term outside the bracket of the denominator is the mutual Coulomb potential between the opposite quarter spirals.

In (2) Δp_{11} and Δp_{110} are, respectively, the self-potentials of a segment in grid form and the solid plate form of the quarter-plate. From [4], they are:

$$\Delta p_{11} = \frac{1}{c_f \varepsilon_0 \sqrt{8\pi W W_s}} \tag{3a}$$

$$\Delta p_{110} = \frac{1}{c_f \varepsilon_0 \sqrt{8\pi W_s^2}} \tag{3b}$$

where the shape factor $c_f = 0.865$ accounting for the charge singularities along only two edges in each segment along the spiral.

The shape factor c_{f1} in Eq. (1), after curve-fitting from a large number of trapezoidal shapes of the quarter-spiral, can be taken as a function of ρ (the *fill ratio* of the spiral inductor due to the hole in Fig. 2a):

$$c_{f1} = 0.90571 + 0.49425 e^{-\rho/0.12253} \tag{4}$$

Eq. (2) is the *far* asymptote of the capacitance of the quarter-spiral as the substrate thickness h becomes very large.

B. The near asymptote of substrate thickness h

When $h \to 0$, that is the substrate is very thin, the capacitance of the square spiral inductor is simply that of parallel plate of conducting spirals, i.e.,

$$C_{1/4near} = \frac{\varepsilon_0 A_{1/4}}{h} \cdot \frac{W}{W_s} \tag{5}$$

C. The synthetic asymptote for A and B

The synthetic asymptote is frequently simply the sum of the two regular asymptotes, *far* and *near*, as observed in many examples of [4]. Hence, for 4 trapezoids, the synthetic asymptote of the square spiral inductor is, through the duality $LC=\mu_0\varepsilon_0$,

$$L = 4\frac{\mu_0\varepsilon_0}{\left(C_{1/4far}^n + C_{1/4near}^n\right)^{\frac{1}{n}}} N^2 (b_1 + b_2)^2 \tag{6}$$

where (b_1+b_2) is the averaged length of the spiral arms in a trapezoid. Even with the power n=1, the synthetic asymptote (6) agrees with far and near asymptotes h converted to L. Obviously, the maximum error is no more than 10% at intermediate values of h as shown in Fig. 3. If now the power n is changed from unity to a numerically matched expression of

$$n = 1.3461 - 0.6592 e^{-0.6139 h/W_s + 0.2918 h/W} \tag{7}$$

The error is reduced again to 2%. The improvement is examined in detail below.

III. Results

To verify the accuracy of the Eq. (6), the numerical method [5] has been chosen for comparison. Table 1 shows the comparison results

TABLE 1
COMPARISON OF THE RESULTS BY SYNTHETIC ASYMPTOTE FORMULA (6) AND NUMERICAL METHOD [5] IN FREE SPACE.

N	d_{out}	W	S	L-num	L-formula	Error
2	300	19	4	1.9984	1.98319	-0.76%
4	300	5	4	9.6603	9.55136	-1.13%
5	171	5.4	1.9	5.6877	5.6239	-1.12%
6	400	24	7	7.1225	7.2288	1.49%
7	300	13	7	7.5329	7.5595	0.35%
8	300	5	4	23.59	23.084	-2.14%
9	230	6.5	5.5	9.5775	9.5561	-0.22%
12	180	3.2	2.1	20.3	19.8286	-2.32%

of square spiral inductors in free space with no ground plane. *W*, *S*, and d_{out} have the unit μm. L-num is by numerical method [5], L-formula is by synthetic asymptote formula (6) in *nH*. The average error is less than 2%.

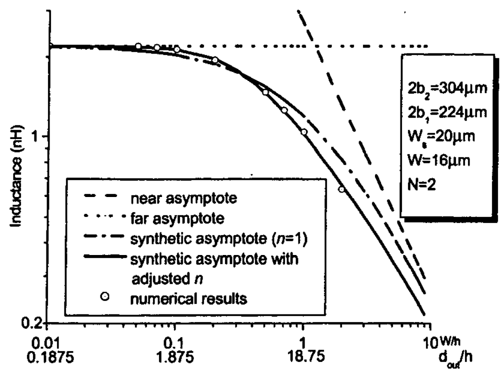

Fig. 3. Comparison of the results by synthetic asymptote formula (6) and numerical method [5].

For different air-substrate thickness, Fig. 3 shows and compares the results calculated by both the synthetic asymptote (6), and the numerical method [5] at 100 MHz. The average error is less than 2%. Also, the regular asymptotes of (2) and (5) converted to *L* are plotted in Fig. 3 to give a physical insight on the approach of the regular asymptotes to the synthetic asymptote. There are two abscissas in Fig. 3, that is: *W/h* and d_{out}/h. This could help the inductor design by giving a comparison on the different dimensions of the spiral with respect to the substrate thickness *h*.

IV. Conclusion

This paper obtained the inductance formulas of square spiral inductor by *duality* and *synthetic asymptote* at low frequency. For high frequency and/or thin substrate, however, the capacitance to ground, the interwinding capacitance, and the air-bridge connecting to the input and/or output become important. They are again derived by the repeated use of the duality relation, the synthetic asymptote and the analytical moment method. The derivations are a little too lengthy to be added in this paper.

It is evident that the above approach of plate partition can be applied to generate an inductance formula for rectangular spiral inductors. With modification, we believe that the approach can also be applied to circular spirals.

References

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